SOME CHARACTERISTIC FEATURES OF DIFFUSION OF A MAGNETIC FIELD INTO A MOVING CONDUCTOR

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ABSTRACT: The study of the diffusion of a magnetic field into a moving conductor is of interest in connection with the production of ultra-high-strength magnetic fields by rapid compression of conducting shells [1, 2]. In [3, 4] it is shown that when a magnetic field in a plane slit is compressed at constant velocity, the entire flux enters the conductor. In the present paper we formulate a general result concerning the conservation of the sum current in the cavity and conductor for arbitrary motion of the latter. We also consider a special case of conductor motion when the flux in the cavity remains constant despite the finite conductivity of the material bounding the magnetic field.

NOTATION

Φ_1, Φ_* is the flux which has diffused into the conductor, Φ_2 is the flux in the cavity, Φ_0 is the sum flux, r is the radius, r_* is the cavity boundary, δ is the thickness of the skin layer, $\delta(r)$ is the delta function of r, t is the time, q is the intensity of the fluid sink, v is the velocity, Φ is the flux which has diffused to a depth larger than r, x is a self-similar variable, φ is the dimensionless fraction of the flux which has diffused to a depth larger than r, φ_* is the fraction of the flux which has diffused into the conductor, σ is the conductivity, c is the electrodynamic constant, R_m is the magnetic Reynolds number, μ is a dimensionless parameter.

$$x = \pi r^2 / qt, \qquad \varphi = \Phi / \Phi_0, \qquad \mu = \sigma q / c^2.$$

§1. **Conservation of the sum flux during diffusion of a magnetic field into a moving conductor.** The problem of diffusion of a magnetic field out of a cavity filled with a moving conductor reduces to solving Maxwell's system of equations in a moving conductor which satisfies certain conditions at infinity and the condition of continuity of the fields at the cavity boundary [3]. It is easy to establish that at low velocities ($v \ll c$) and high conductivities the sum of the flux $\Phi_1(t)$ which has diffused into the conductor and the flux $\Phi_2(t)$ in the cavity is constant at any instant, i.e., that

$$\Phi_1(t) + \Phi_2(t) = \Phi_0,$$

provided the conductor is of infinite extent and that the field is always zero at infinity. This is a simple consequence of the law of induction, since under these conditions the circulation integral of the magnetic field intensity over an infinite contour becomes zero, with the result that the derivative of the magnetic flux through such a contour with respect to time also equals zero. This means that the sum flux in the cavity and conductor is conserved.

§2. **A special case of conductor motion. Constancy of the flux in a cylindrical cavity.** There are special cases of conductor motion when not only the sum flux, but also the flux in the cavity, remain constant despite finite conductor conductivity. There is no flux leakage from the cavity when the field in the conductor varies in the same way with time due to its motion and to the diffusion of the flux which has penetrated into it, as does the field in the cavity due to the variation of cavity size alone. In the absence of flux leakage from a cylindrical cavity, the field in the cavity is $H \sim 1/r^2$ and the field in the conductor $H \sim 1/r\delta$, where r is the radius of the cavity and δ is the thickness of the skin layer. This implies that the above effect can occur if $r \sim \delta$, i.e., if the variation of cavity size with time is the same as the time-dependent increase in skin layer thickness. If $r \sim (t)^{1/2}$, then the thickness of the skin layer is also proportional to $(t)^{1/2}$, since diffusion of the field into the conductor extends to a depth proportional to $(t)^{1/2}$, and since transfer of the field with the moving conductor is in this case also proportional to $(t)^{1/2}$.

The condition $r \sim (t)^{1/2}$ for an incompressible conducting liquid corresponds to the case of a single source or sink of constant intensity q. Let the sink lie at intensity, i.e., let the cavity expand. Then

$$V = q / 2\pi r, \qquad r_*^2 = qt / \pi.$$

The equation of diffusion of the magnetic flux in this case is

$$\frac{\partial\Phi}{\partial t} + \frac{q}{2\pi r}\frac{\partial\Phi}{\partial r} = \frac{c^2}{4\pi\sigma} r \frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\Phi}{\partial r} \quad \left(\Phi = 2\pi\int_r^\infty rH\,dr\right).$$

Here Φ is the flux which has diffused to a depth larger than r by a certain instant. The conditions of conservation of the flux and continuity of the magnetic field,

$$\Phi(r, t)|_{r=r_*} = \Phi_*(t), \qquad \Phi_0 - \Phi_*(t) = -\frac{r}{2}\frac{\partial\Phi}{\partial r}\Big|_{r=r_*},$$

must be fulfilled at the cavity boundary.

If we assume that the flux at the initial instant is equal to zero only at the axis, i.e., if

$$\Phi(r, 0) = \Phi_0\delta(r),$$

then the problem is self-similar and reduces to finding (for $1 \le x < \infty$) the solution of the equation

$$\mu^{-1}x\varphi'' = (1 - x)\varphi' \qquad (1 \leqslant x < \infty) \tag{2.1}$$

which satisfies the conditions

$$\varphi = 0 \quad \text{as} \quad x \to \infty, \quad \varphi|_{x=1} = \varphi_*, \quad 1 - \varphi_* = -(x\varphi')|_{x=1}. \tag{2.2}$$

Here we have introduced the self-similar variable $x = \pi r^2/qt$ and the dimensionless flux in the conductor $\varphi = \Phi/\Phi_0$. The parameter

$$\mu = \sigma q / c^2_2$$

is related to the magnetic Reynolds number R_m [5] by the expression $\mu = R_m/2$. The boundary of the cavity is defined by the condition $x = 1$. The condition of continuity of the magnetic field at the cavity boundary (condition (2.2)) implies that $\varphi_* =$ const, i.e., that the fraction of the flux which has diffused into the conductor remains constant during expansion of the cavity, as a result of which the flux in the cavity is also constant.

The solution of Eq. (2.1) which satisfies the condition at infinity is

$$\varphi = C\int_x^\infty x^\mu e^{-\mu x}\,dx.$$

The conditions at the cavity boundary imply that

$$C = \varphi_*\left(\int_1^\infty x^\mu e^{-\mu x}\,dx\right)^{-1}, \qquad \frac{1}{\varphi_*} = 1 + e^{-\mu}\left(\int_1^\infty x^\mu l^{-\mu x}dx\right)^{-1}. \tag{2.3}$$

From (2.3) we readily infer that

$$\varphi_* = 1 - \mu^{1+\mu}(1 + 0(\mu)) \qquad \text{for} \quad \mu \ll 1.$$

Using the Laplace method [6] to estimate the integral in (2.3), we obtain

$$\varphi_* \approx \sqrt{\frac{\pi}{2}}\frac{1}{\sqrt{\mu}} \qquad \text{for} \quad \mu \gg 1.$$

The above example shows that in our special case of conductor motion the flux in the cavity can remain constant, differing from the original flux by quantities on the order of $1/(\mu)^{1/2}$ for high velocities and having a value on the order of μ^{1+u} for low velocities.

The above effects have direct analogs in the corresponding problems of heat conduction.

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